Homework 4 - Polarization

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1. Assume a 1 Watt radio tower emitting at 10 kHz, radiates uniformly in 4π steradians. At a distance of 100 meters, determine values for E (V/cm), intensity (W/cm^2), photon flux (#/cm^{2}sec), and photon density per cubic wavelength (i.e.Volume = λ^3). Repeat also for a 1-Watt source at 550-nm wavelength. What do your values say about the using the particle and wave views for EM radiation?

2. The Sellmeier formula for a crystal is (λ_o in microns):

$$n^2 = 2.7359 + \frac{0.01878}{\lambda_o^2 - 0.01822} - 0.01345\,\lambda_o^2$$

   (a) Make a graph of n verses frequency across the visible spectrum.
   (b) What is Brewster's angle at 0.5 µm.
   (c) What is the reflectance, R, for s and p polarized light at this angle.
   (d) Determine the phase and group velocities at 0.5 µm in the crystal.

3. What minimum number of Brewster Plate windows, having index of refraction = 1.75, will polarize a randomly polarized source to at least 100:1 intensity ratio?

4. Circularly polarized light is incident at 45^o on the internal surface of a 90-45-45 prism (n = 1.5) in the geometry shown on page 71 of the viewgraphs (Phase-Shift for internal reflections). The incident field components are:

$$E_x = E_o \cos \omega t \qquad \text{(parallel polarization)}$$

$$E_y = E_o \sin \omega t \qquad \text{(perpendicular polarization)}$$

   (a) Determine the state of polarization upon reflection, carefully sketch the electric field in the y - z plane, show the temporal sense of rotation, and give the electric field amplitudes and orientation along the semi-major and semi-minor axes. To solve this problem, cast the fields into the form of cos (A + B) and cos (A - B) before rotating the y - z axis by a 45^o angle.
   (b) The evanescent wave for parallel polarization has an electric field that can be expressed as:

$$E = E_o(ae^{i\phi})\, e^{-(\beta_x x+\beta_y y+\beta_z z)}\;\; e^{i(k_x x+k_y y+k_z z-\omega t)}$$

   where a, ϕ, β_x, β_y, β_z, k_x, k_y, and k_z are real values to be determined. Express k_i and β_i values as functions of k_o (= $2\pi/\lambda_o$).
   (c) How far does the evanescent wave reach past the prism surface to the 1/e field amplitude of the surface field.

5. Linearly polarized light consisting of equal "parallel" and "perpendicular" polarization components is incident at angle, θ_i, onto a glass (n = 1.5) to air (n = 1.0) boundary.
   (a) State the Brewster and critical angles for these internal reflections.
   (b) State the polarization type (linear, circular, elliptical, or unpolarized) following reflection at the following internal angles.

   (i) $\theta_i = 15^o$ (ii) θ_i = Brewsters angle (iii) $\theta_i = 35^o$
   (iv) θ_i = Critical angle (v) $\theta_i = 75^o$

6. Prove that the spherical harmonic wave function

$$\frac{1}{r}\, e^{i(kr-\omega t)}$$

   is a solution of the three-dimensional wave equation, where $r = (x^2 + y^2 + z^2)^{1/2}$. The proof is easier if spherical coordinates are used.

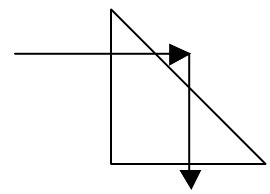
7. Derive the formulas

$$u_g = u - \lambda \frac{du}{d\lambda} \qquad \text{and} \qquad \frac{1}{u_g} = \frac{1}{u} - \frac{\lambda_0}{c}\frac{dn}{d\lambda_0}$$

8. Find the critical angle for internal reflection in water (n = 1.33) and diamond (n = 2.42).

9. Find the reflectance for both TE and TM polarizations at an angle of incidence of 45 degrees. What is the Brewster angle for external reflection?

10. A beam of light is totally reflected in a 45-90-45-degree glass prism (n = 1.5) as shown below. The wavelength of the light is 500 nm. At what distance from the surface is the amplitude of the evanescent wave 1/e of its value at the surface? By what factor is the intensity of the evanescent wave reduced at a distance of 1 mm from the surface?



11. State the type of polarization (linear, circular, elliptical, etc.) for each case below (sense of rotation, field orientation, amplitudes are not required):

(a) $\vec{E} = E\,\hat{x}\,e^{i(kz-wt)} + E'\,\hat{y}\,e^{i(kz-wt+f)}$

(i) $E = E'$, $\phi = \pi/2$ (ii) $E = E'$, $\phi = \pi/4$ (iii) $E \neq E'$, $\phi = 0$

(iv) $E \neq E'$, $\phi = \pi/2$ (v) $E = E'$, ϕ = random

(b) Assume linear polarized light is incident on a glass-to-air boundary (from glass to air) with equal strength of the parallel and perpendicular polarization fields. Assume n = 1.5 for glass.

(i) reflection for $\theta_i = 15^o$
(ii) reflection for $\theta_{Brewster}$
(iii) reflection for $\theta_i = 45^o$
(iv) transmission for $\theta_i = 45^o$
(v) state a reason for your answer in (iv)

12. State the factors contributing to dispersion in a single-mode fiber.

13. A multimode fiber with n_1 = 1.486 and n_c = 1.441 is required for a local area network running at 55 MHz. Estimate the maximum possible length of the network?

14. (a) Use diagrams and calculations to show `precisely` how linearly polarized light at 0.6-μm wavelength can be converted to circularly polarized light with calcite given n_e =1.486 and n_o =1.658. Specify any parameters you feel are necessary.

(b) Use Fresnel's equations to calculate the total transmittance (both surfaces) for the e- and o-rays in part (a). What discrepancy arises?

Homework #4 Intr'n to Optical Eng'g Sol'ns

P. Herman

1. @ $\nu = 10$ kHz $I = 7.96\times10^{-10}$ W/cm² $E = 7.75\times10^{-4}$ V/cm
   $F = 1.2\times10^{20}$ ph/cm²-s $\rho = 1.08\times10^{29}$ ph/λ^3

   @ $\lambda = 550$ nm I, E – the same
   $F = 2.2\times10^{9}$ ph/cm²-s $\rho = 1.22\times10^{-14}$ ph/λ^3

2. (a)

λ (μm)	ν (Hz)	n
0.7	4.3×10^{14}	1.664
0.5	6×10^{14}	1.677
0.4	7.5×10^{14}	1.693

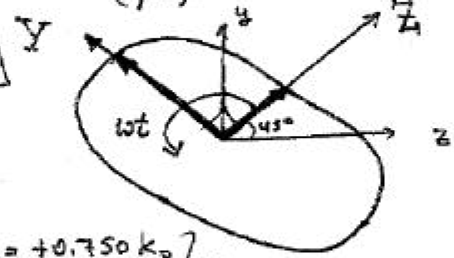
(b) $\theta_{Br} = 59.19°$

(c) $R_p^{\parallel} = 0\%$ $R_s^{\perp} = 22.6\%$

(d) $\frac{1}{u_{gr}} = \frac{1}{u_\phi} - \frac{\lambda_0}{c}\frac{dn}{d\lambda_0}$ $\left\{ \begin{array}{l} u_\phi = 1.789\times10^{10} \text{ cm/s} \\ u_{gr} = 1.733\times10^{10} \text{ cm/s} \end{array} \right\}$

3. $\theta_{Br} = 60.26°$; $R_\perp = 25.78\%$; $R_\parallel = 0\%$; $\left(\frac{T^{\parallel}}{T^{\perp}}\right)^{2n} = 100 \Rightarrow n = 7.697 \rightarrow \underline{\underline{8}}$

4. follow notes in lab/class – work out dommus



$\frac{\phi_x - \phi_\perp}{2} = 18.435°$

elliptical $E_z^R = 0.632\,E_0$, $E_Y^R = 1.265\,E_0$

(b) $k_x = -0.750\,k_0$; $k_y = 0$; $k_z = +0.750\,k_0$; $k_0 = 2\pi/\lambda_0$
$\beta_x = +0.250\,k_0$; $\beta_z = +0.250\,k_0$; $\beta_y = 0$
$t_\parallel = 2.121\,e^{-0.643 i} = a e^{i\phi} \Rightarrow a = 2.121$; $\phi = -0.643$ radians

(c) $\beta^{\perp}_{wall} = \frac{2\pi}{\lambda_0}(0.354) \Rightarrow e^{-\beta^{\perp}_{wall} d} = e^{-1} \Rightarrow d = 1/\beta^{\perp}_{wall} = \underline{\underline{0.450\,\lambda_0}}$

5. (a) $\theta_{Br} = 33.69°$ $\theta_{cr} = 41.81°$
   (b) (i) $\theta_1 = 15°$ $r^\parallel, r^\perp$ are real $\Rightarrow$ linear
   (ii) $r^\parallel = 0$, $r^\perp$ real $\Rightarrow$ linear
   (iii) $r^\parallel, r^\perp$ real $\Rightarrow$ linear
   (iv) $r^\parallel, r^\perp$ "real" $\Rightarrow$ linear
   (v) $\theta_1 = 75°$ $r^\parallel, r^\perp$ have phase $|r| e^{i\phi} = e^{i\phi}$ } elliptical

#8, 9 {water $\theta_{cr} = 48.75°$ $R_\perp = 5.23\%$ $R_\parallel = 0.27\%$
{diamond $\theta_{cr} = 24.4°$ $R_\perp = 28.3\%$ $R_\parallel = 8.01\%$

#10 (a) 0.00033 mm (b) 10^{-2570} (this is zero)

Homework #4 ECE426 Solutions Continued

11a) (i) circular (ii) elliptical (iii) linear (iv) elliptical (v) Random

b) (i) linear (ii) linear (iii) FTIR with $\phi_\perp \neq \phi_{\parallel} \Rightarrow$ elliptical

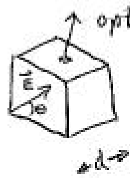
(iv) elliptical evanescent (v) since $\phi_{\parallel} \neq \phi_\perp$ and $t_{\parallel} \neq t_\perp$

12 material dispersion – group velocity is λ-dependent

waveguide dispersion – different λ bounce at different θ_{mode} inside waveguide arriving at different times.

13 $R = \frac{1}{2\Delta t} = \frac{c}{2Ln_f}\left(\frac{n_f}{n_c} - 1\right)^{-1} = 55\,MHz \Rightarrow L = 59$ meters

14 (a)



$\theta = 45^0 \Rightarrow E_x = E_y = \frac{E_0}{\sqrt{2}}$

$\Lambda = \frac{\pi}{2} = \left|(k_e d - k_0 d)\right| = \frac{2\pi}{\lambda_0}(n_e - n_0)\, d \Rightarrow \boxed{d = 0.87\,\mu m}$

(b) $R_e = \left(\frac{n_e - 1}{n_e + 1}\right)^2 = 3.82\% \Rightarrow T_e = 96.18\%$

$R_o = \left(\frac{n_o - 1}{n_o + 1}\right)^2 = 6.13\% \Rightarrow T_o = 93.87\%$

Pol'n Ratio after 2 surfaces $= \left(\frac{T_e}{T_o}\right)^2 = 1.05$

∴ slight elliptical beam