High-speed laser modulation beyond the relaxation resonance frequency limit

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Abstract: We propose and show that for coupling modulated lasers (CMLs), in which the output coupler is modulated rather than the pump rate, the conventional relaxation resonance frequency limit to the laser modulation bandwidth can be circumvented. The modulation response is limited only by the coupler. Although CMLs are best suited to microcavities, as a proof-of-principle, a coupling-modulated erbium-doped fiber laser is modulated at 1 Gb/s, over 10000 times its relaxation resonance frequency.

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OCIS codes: (140.3460) Lasers; (230.0230) Optical devices; (140.4780) Optical resonators.

References and links

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1. Introduction

Over the past several decades, substantial efforts have been undertaken to increase the relaxation resonance frequency of lasers to increase the maximum bandwidth at which lasers can be directly modulated; for example, through engineering the quantum confinement in the active medium [1–8], utilizing injection locking techniques [8–10], and leveraging cavity quantum electrodynamics effects [11, 12]. The direct modulation of a laser involves modulating a laser parameter, most often the pump rate, to impart information onto an optical carrier. It is commonly accepted that relaxation oscillations, which arise from the coupling between the atomic population in the gain medium and the photon density in the optical cavity, limit the modulation bandwidth of lasers [13, 14]. The relaxation resonance frequency, $f_R$, decreases with increasing resonator quality factor, $Q$, and increases with the cavity photon density. In addition, as the pump rate is modulated, the gain and refractive index inside the laser cavity are modified; hence, the laser output is also unavoidably chirped [8, 13, 14]. For typical semiconductor lasers, $f_R$ is about 1-10 GHz [8, 13, 14], and for erbium-doped fiber lasers (EDFLs), $f_R$ is roughly $< 1$ MHz [15].

In this Letter, we propose and show that for a coupling-modulated laser (CML), in which the output coupler is modulated rather than the pump, the laser modulation bandwidth can be orders of magnitude larger than $f_R$. Moreover, CMLs can circumvent the conventional chirp limitations and trade-offs between the laser threshold, drive power, and modulation bandwidth. In addition to large modulation bandwidths, CMLs can also achieve high extinction ratios with low drive powers, making them promising for chip-scale optical interconnects and networks. Although the technological benefits of CMLs are most obvious using microcavities, to illustrate a CML inherits the coupler modulation response, we demonstrate a coupling-modulated EDFL at 1 Gb/s, over 10000 times its relaxation resonance frequency.

2. Principle of operation

2.1 Intuitive explanation

A CML implemented with a ring cavity is shown in Fig. 1(a), where the cavity is integrated with a 1x2 variable coupler. $\kappa$ and $\sigma$ are the field cross- and through-coupling of the coupler, respectively. The basic principle of operation can be understood simply. If we modulate the coupler and keep the pumping inside the cavity constant, the output power, $P_{out}(t)$, is

$$P_{out}(t) = |\kappa(t)|^2 P_{in}(t),$$

(1)
where $P_{in}(t)$ is the circulating power inside the resonator. The output depends on two effects: 1) The instantaneous gating $|\kappa(t)|^2$ of light as it exits the resonator, and 2) the modulation of the circulating power $P_{in}(t)$, which is influenced by the memory of the resonator and the response of the gain medium. If $P_{in}(t)$ is approximately constant, the laser modulation is only limited by the coupler. Since $P_{in} > P_{out}$ in optical cavities, only a small modulation in $|\kappa(t)|^2$ is required. Compared to using the coupler as an external modulator, the change in $|\kappa(t)|^2$ in a coupling-modulated laser is reduced by $P_{in}/P_{out}$. Hence, coupling modulation leverages the large intracavity power to increase the modulation efficiency. This modulation is distinct from Q-switching and mode-locking, since the circulating power is not significantly perturbed.

The condition at which $P_{in}(t)$ remains essentially static is at modulation rates much greater than $f_R$, when the photon density can no longer respond to changes in the cavity. At low modulation frequencies (less than and near $f_R$), $P_{in}(t)$ can respond strongly to changes in the output coupling ratio, which can cause significant output distortion. The range of modulation frequencies over which distortion is significant can be minimized by reducing $f_R$, for example, by increasing the cavity finesse. Therefore, at sufficiently high modulation rates, coupling-modulation decouples the optical modulation from the intrinsic response of the gain medium and the cavity. The laser instead inherits the modulation characteristics of the coupler [16, 17]. The coupler, for example, can be a Mach-Zehnder interferometer (MZI) [16, 18–20], which can possess large modulation bandwidths [21] and be chirp-free [22].

### 2.2 Rate equation analysis

To mathematically illustrate the observations in the previous section, we model the intracavity power using rate equations. We assume a microcavity implementation of the laser, such that the modulation frequency, for practical purposes, does not exceed the cavity free spectral range (FSR). Neglecting spontaneous emission, imperfect waveguide confinement, and the spatial dependency of the intracavity field, the laser dynamics can be modeled by the following rate equations [13]:

\[
\frac{dN}{dt} = R_{pump} - \frac{N}{\tau_c} - \frac{v_g a(N-N_{tr})P_{in} \lambda}{Vhc},
\]

\[
\frac{dP_{in}}{dt} = v_g a(N-N_{tr})P_{in} - \left[ v_g (\alpha - \frac{\ln(\sigma \gamma)}{\tau}) \right] P_{in},
\]

where $N$ is the carrier (i.e. atomic) concentration of the upper laser level, $N_{tr}$ is the transparency carrier concentration, $R_{pump}$ is the pump rate, $v_g$ is the group velocity of the circulating light, $a$ is the differential gain, $V$ is the mode volume, $\lambda$ is the laser mode...
wavelength, \( \tau_c \) is the carrier lifetime, \( \tau \) is the cavity round-trip time, and \( \alpha \) is the loss per length in the cavity. If the pump rate is modulated, \( R_{\text{pump}}(t) \), the modulation bandwidth is roughly limited to the relaxation resonance frequency, \( \omega_R = 2\pi f_R = \{v_c\alpha\tau P_{in,0} [v_c\alpha\tau - \ln(|\sigma|^2)|/(Vhc)]^{1/2} \}, \) where \( P_{in,0} \) is the bias intracavity power [13].

Next, we perform a small-modulation-signal analysis of the coupling modulation, so \( R_{\text{pump}} \) is constant and \( \kappa(t) = \kappa_0 + \omega c(t) \), \( P_{in}(t) = P_{in,0} + \varepsilon P_{in}'(t) \), \( N(t) = N_0 + \varepsilon N'(t) \), where \( \varepsilon \ll 1 \). We assume \( |\sigma(t)|^2 + |\kappa'(t)|^2 = 1 \) and \( \ln(|\sigma(t)|^2) \approx \ln(|\kappa'(t)|^2) \), which is an excellent approximation for small output coupling (i.e. \( \kappa(t)^2 < 0.1 \)). Taking the Fourier transform of Eq. (2), solving for \( P_{in}(\omega) \) to \( \Omega(\omega) \), where \( \omega \) is the modulation frequency, and substituting into Eq. (1), we find

\[
P_{\text{out}}(\omega) = |\kappa_0|^2 P_{in,0}\delta(\omega) + \varepsilon P_{in,0} [\kappa_0*\kappa^*(-\omega) + \kappa_0*\kappa^*(\omega)] \left[ 1 + M(\omega) \right], \quad (3a)
\]

\[
M(\omega) = \frac{|\kappa_0|^2 P_{in}'(\omega)}{|\kappa_0*\kappa^*(-\omega) + \kappa_0*\kappa^*(\omega)|^2} = \frac{|\kappa_0|^2}{\tau} \left[ i\omega + \left( 1/\tau_c + \omega_R^2\tau_{ph} \right) \right]. \quad (3b)
\]

The first term in Eq. (3a) is a constant offset and the second term is the modulation of the output power. The \( \kappa_0*\kappa^*(-\omega) + \kappa_0*\kappa^*(\omega) \) factor is the instantaneous modulation of the output power by the coupler, and \( M(\omega) \) is a measure of the modulation response of the intracavity field. Also, \( \tau_{ph} = v_c\alpha - \ln(|\sigma|^2)\tau_c \) is the photon cavity lifetime.

Equation (3) shows two distinct bands of modulation frequencies which are depicted in Fig. 1(b): 1) For modulation rates, \( \omega \), near and less than \( \omega_R, \left| M(\omega) \right| \) is significant and not flat with respect to \( \omega \), which implies significant output distortion. This distortion from the intracavity field response is equivalent to the memory effect distortion described in [16], and the width of this distortion band can be decreased by lowering \( \omega_R \) [13]. 2) For modulation rates \( \omega \gg \omega_R, \left| M(\omega) \right| \) approaches 0, and therefore, \( P_{in}(t) \approx P_{in,0} \).

Therefore, if we choose modulation signals with small spectral overlap with the memory effect distortion band, the circulating power remains essentially constant; hence, Eq. (3a) becomes

\[
P_{\text{out}}(t) \approx |\kappa(t)|^2 P_{in,0}, \quad (4)
\]

and the laser inherits the modulation response of the coupler with an enhancement by \( P_{in,0} \).

The approximation of a constant circulating power in Eq. (4) is most appropriate for high finesse cavities, i.e. small, high-\( Q \) cavities, at a fixed intracavity power. Since \( \omega_R \) decreases with increasing \( Q \), the band of frequencies over which memory effect distortion is significant is reduced. High-\( Q \) cavities have the additional benefit of a large \( P_{in,0} \) at a given pump rate, which leads to a large output modulation for a small coupling modulation. Also, the cavity FSR should be large, so that the modulation frequencies do not have significant overlap with integer multiples of the FSR. Memory effect distortion can be severe at multiples of the FSR, as the generated modulation sidebands are the highest at the resonant frequencies. Therefore, coupling modulated lasers (CMLs) are suited to microcavity implementations.

Microcavity CMLs may use standing wave or traveling wave cavities incorporating a directional coupler or Mach-Zehnder interferometer (MZI) output coupler that can be modulated. Depending on the size and geometry of the microcavity CML, spontaneous emission and gain saturation effects may significantly influence the output noise and memory effect distortion.

### 3. Coupling-modulated fiber laser

As a proof-of-concept demonstration that a CML inherits the modulation response of the coupler at high modulation frequencies and to clearly delineate the contributions of the coupler to the modulation response, we constructed a coupling-modulated erbium-doped fiber laser (EDFL). The laser is a ring cavity integrated with a lithium niobate (LiNbO\(_3\)) dual output modulator (Avanex PowerLog AM-1) as an output coupler. The 3-dB modulation
bandwidth of the coupler was about 800 MHz. Figure 2(a) shows the schematic of the laser. The gain section was 15 m of erbium-doped fiber (EDF, Lucent Ofs HE980) pumped with a 980 nm laser diode. About 25% of the pump laser power was coupled into the EDF. An unpumped section of EDF coupled to a fiber Bragg grating (FBG) acted as a narrow-band tracking filter [23, 24] to prevent mode-hopping and maintain single-mode operation at 1532 nm. The reflection spectrum of the FBG was centered at 1532 nm and had a full-width at half-maximum of 0.25 nm. FC/APC connectors and fusion splicing were used for all connections in the laser. The polarization controller was adjusted to minimize the loss through the LiNbO$_3$ coupler and maximize the modulation efficiency. The free spectral range (FSR) of the cavity was about 7 MHz, and the round-trip loss without the unpumped EDF was about 10 dB. Drift in the splitting ratio of the LiNbO$_3$ coupler was carefully monitored throughout the experiment to ensure significant changes in the bias point did not occur during measurements.

3.1 Steady-state laser output and relaxation resonance frequency

First, we characterized the steady-state laser output. Figure 2(b) shows the output power versus coupled pump power ($P_{\text{pump}}$) for two of the bias power-coupling strengths, $|\kappa|^2$, discussed in this letter. We verified single-mode operation using a high finesse scanning Fabry-Perot interferometer (Newport Supercavity) with a FSR of 6 GHz and a resolution of 1.2 MHz. A sample laser spectrum for $|\kappa|^2 = 0.5\%$ and $P_{\text{pump}} = 75$ mW is shown in Fig. 2(c). The absence of extraneous peaks indicates that the laser was single-mode.

To determine $f_R$, we modulated the current of the pump laser diode with 100 µs pulses and measured the ringing frequency in the output intensity. For the bias conditions used in this experiment, $f_R < 100$ kHz. Figure 2(d) shows an example of the output, where $f_R$ is approximately 70 kHz.

3.2 Small signal modulation

Next, we measured the small-sinusoidal-signal modulation response of the laser using a 40 GHz photodetector (Newport D-8ir) and a vector network analyzer (Agilent 8753ES).
comparison, we performed a similar measurement of the LiNbO$_3$ coupler by itself using a 6 mW optical input at 1532 nm from a tunable laser. Figure 3(a) shows the small-sinusoidal-

![Graph](image)

Fig. 3. (a) Small-sinusoidal-signal response of the coupling-modulated laser ($P_{pump}=75$ mW) and the coupler modulator for $|\kappa_0|^2=0.5\%$ between 30 kHz to 5 GHz. The inset of (a) shows the output intensity with 100 MHz coupling modulation. (b) A high resolution measurement of the coupling modulation response of the laser measured in (a).

signal responses of the laser and the coupler, which are nearly identical, confirming that the laser modulation roll-off is essentially due to the coupler. The output at specific modulation frequencies was directly detected to inspect the extent of any distortion. The inset of Fig. 3(a) shows an example at 100 MHz, which exhibits minimal harmonic distortion. Figure 3(b) shows a measurement of the small-sinusoidal-signal coupling modulation response of the laser over a 20 MHz range. The spectral resolution of Fig. 3(b) is much higher than Fig. 3(a), and dips are observed in the modulation response at the resonance frequencies of the laser cavity.

3.3 1 Gb/s modulation

Finally, we modulated the laser with 1 Gb/s bit patterns. The electrical bit patterns were generated by a pattern generator (HP70841B) and the laser output was measured using a digital communications analyzer (Agilent 86100C, 86106B module). Figure 4 shows the eye diagrams and corresponding bits for the laser and the coupler by itself for a $2^7$-1, non-return-to-zero (NRZ) binary pseudo-random bit sequence (PRBS) modulation pattern. The drive voltage on the modulator was kept at 500 mV$_{pp}$. The modulator $V_z$ was $> 8$ V at 1 GHz. We compared two operation points: $|\kappa_0|^2=1\%$ with a 2% peak-to-peak $|\kappa|^2$ modulation and $|\kappa_0|^2=$
Fig. 4. Comparison of the coupler modulator with the EDFL for 1 Gb/s, NRZ, 2\(^{7}\)-1 PRBS modulation (\(P_{\text{pump}} = 75 \text{ mW}\)). 40 bits of the PRBS pattern for (a) the coupler drive voltage, (b) EDFL output power with \(|\kappa_0|^2 = 1\%\), and (c) EDFL output power with \(|\kappa_0|^2 = 7\%\). (d) Coupler and (e) EDFL eye patterns with \(|\kappa_0|^2 = 1\%\). (f) Coupler and (g) EDFL eye patterns with \(|\kappa_0|^2 = 7\%\). Coupler eye patterns are normalized to match the scale of the EDFL eye patterns. Due to the detector noise, each bit trajectory was averaged 512 times.

Fig. 5. (a) The envelope of the output obtained by undersampling the EDFL output with magnified views of the beginning and end of the pattern. (b) The eye patterns for the coupler and EDFL for the last 2\(^{15}\) bits. The bit patterns were averaged over 300 repetitions in (a) and 18 repetitions in (b).

7\% with a 6\% peak-to-peak \(|\kappa|^2\) modulation. The coupling bias, defined from the average of the maximum and minimum output powers, shifted from 0.5\% to 1\% at the same bias voltage as the small-signal and steady-state measurements, because \(|\kappa(t)|^2\) varies quadratically with the modulation voltage at \(|\kappa_0|^2 \approx 0\).

For \(|\kappa_0|^2 = 1\%\), the eye patterns of the coupler and the EDFL [Figs. 4(d), (e)] are nearly identical, which further confirms the laser modulation was determined by the coupler. The distortion in the laser eye pattern can be attributed to fluctuations in \(P_m\). The extinction ratio of the laser modulation is 10 dB. At 1 Gb/s, the modulation rate of the laser is over 10000 times its relaxation resonance frequency.

For \(|\kappa_0|^2 = 7\%\), the output is severely distorted [Fig. 4(c)], and the eye pattern of the laser is closed [Fig. 4(g)] even though the coupler eye pattern exhibits little distortion [Fig. 4(f)]. The distortion is caused by the larger fluctuations in \(P_m\) relative to its steady-state value. Due to the low finesse of the EDFL cavity, \(|\kappa_0|^2\) could not simultaneously maximize the laser output power and minimize the disturbance of \(P_m\). This limitation is overcome in high finesse
cavities, since the optimal output coupling that maximizes $P_{out}$ decreases with intrinsic cavity losses [14]. High finesse cavities have the added benefit of low laser thresholds [25].

3.4 Turn-on transients

To observe the laser transient characteristics, we modulated the coupler with a $2^{19}$ random bit pattern with $|\kappa_0|^2 = 1\%$ and a 2% peak-to-peak $|\kappa|^2$ modulation. The laser was allowed to regain a steady-state for $1.57 \text{ ms} (>> 1/f_R)$ between each repetition of the bit pattern. The envelope of the EDFL at long time scales [Fig. 5(a)] shows that the modulation initially triggers relaxation oscillations that decay to a near constant envelope. Figure 5(b) shows the eye patterns for the last $2^{15}$ bits for the EDFL and the coupler by itself are nearly identical, consistent with the PRBS modulation in Fig. 4. The characteristic time for $P_{in}(t)$ to reach a steady-state, essentially constant value with the modulation is the lifetime of the relaxation oscillations. For CMLs fabricated in compound semiconductor materials, the time required to reach steady-state could be much shorter, of the order of 100 ps.

4. Conclusions

In summary, we have proposed and demonstrated a coupling-modulated laser (CML), which circumvents the traditional relaxation resonance frequency bandwidth limitation. Although CMLs are suited to microcavity implementations, our experiment with an erbium fiber laser clearly delineated the contributions of the coupler and the material response to the coupling modulation. At sufficiently high modulation rates, the output modulation is decoupled from the gain medium and limited by the coupler. Compared to external modulation, coupling-modulation increases the modulation efficiency by a factor proportional to the intracavity power. The coupler can be based on other types of modulators to achieve phase modulation, chirp-free intensity modulation, and quadrature modulation [26,27]. These principles are equally applicable to standing-wave lasers.

Acknowledgments

The support of the Natural Sciences and Engineering Research Council of Canada, the Canada Foundation for Innovation, and the Ontario Ministry of Research and Innovation is gratefully acknowledged. We thank Prof. Li Qian and Dr. Wen Zhu for lending us the detector and pulse pattern generator.